would require an intercept of about 0.3. This value is in sharp conflict with the higher intercepts of 0.7 and 0.8, which we have favored here.

Recently, Abolins *et al.*[8] have reported evidence for a resonance at 1.22 BeV that may have the same spin and quantum numbers as the ρ. If so, there would be a second ρ trajectory with a smaller intercept at $t=0$ than the first. The combined result might be an "average" intercept of about 0.3 as required by the experiment of Palevsky *et al.*[7] (If the resonance at 1.22 BeV has spin 3 and is the recurrence of the ρ, it would roughly fit into our solution with the intercept of 0.5.)

[8] M. Abolins, R. L. Landers, W. A. W. Mehlhop, Nguyen-huu Xuong, and P. M. Yager, Physics Department, University of California at San Diego, La Jolla, California, September 1963 (unpublished).

ACKNOWLEDGMENTS

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Interrelations Among Dissymmetries in SU_3 Supermultiplets

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The mass splittings within the (unitary) supermultiplets of the strongly-interacting particles are studied in the octuplet model of SU_3 symmetry. Using arguments based on the requirement of dynamical self-consistency, qualitative properties of the first-order perturbations in masses and coupling constants are studied in the ladder approximation. Further evidence for the origin of the particular type of dissymmetry leading to the Gell-Mann–Okubo mass formula and to isotopic spin symmetry is gotten through the relations between the dynamical effects in different supermultiplets. The observed mass splittings in the pseudoscalar meson, vector meson, and the $\frac{1}{2}(+)$ baryon octuplets, as well as in the $\frac{3}{2}(+)$ baryon decuplet, are compatible with the general features expected from first-order perturbations. Some higher order perturbations in the $\frac{3}{2}(+)$ decuplet are also discussed.

I. INTRODUCTION

THE concept of the strongly interacting particles as self-consistent bound states has been applied in an earlier paper[1] to a study of the deviations from SU_3 symmetry in a model containing only vector mesons. We shall discuss here, on the same basis, a more realistic model in which several kinds of particles enter. We shall make use of the following points which were discussed in detail in our previous work. We begin with a self-consistent set of particles which incorporates full symmetry, so the search for additional sets of self-consistent particle masses can be carried out through calculations which make use of the techniques of ordinary perturbation theory. If a first-order perturbation having a particular transformation character approximately reproduces itself, there will be another self-consistent set of particles with a small dissymmetry of the given type; the magnitude of this dissymmetry is then fixed by the self-consistency requirement, but depends on the higher order terms. Moreover, if this self-generating dissymmetry corresponds to a (1,1) (or 8-fold) tensor, general characteristics of the higher order terms imply the necessary maintenance of isotopic spin symmetry. In the vector-meson model, our estimates of the effects of perturbations did indeed favor the (8) dissymmetry, and we shall show that the same result is obtained here as well.

It is sufficient for us to consider here only perturbations which retain isotopic spin invariance, because in first order only the SU_3-multiplet character of the perturbation is relevant, and because of the fact that a self-consistent (8) perturbation necessarily retains SU_2 invariance. The normalized charge-independent mass deviations in the baryon octuplet are listed in Tables I and II for the possible dissymmetries.

Our attention in this paper will be given mainly to the baryon states, but we first remark on the inter-

TABLE I. Mass deviations in octuplets.

Y, T	$1, \frac{1}{2}$	$0, 1$	$0, 0$	$-1, \frac{1}{2}$	Normalization
(8)+	1	−2	2	1	$2\ (5)^{1/2}$
(8)−	−1	0	0	1	2
(27)	3	−1	−9	3	$2\ (30)^{1/2}$

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[1] R. E. Cutkosky and Pekka Tarjanne, Phys. Rev. **132**, 1888 (1963), hereafter cited as I. The present paper contains additional references.

TABLE II. Mass deviations in decuplets.

Y, T	$1, \frac{3}{2}$	$0, 1$	$-1, \frac{1}{2}$	$-2, 0$	Normalization
(8)	-1	0	1	2	$(10)^{1/2}$
(27)	3	-5	-3	9	$(210)^{1/2}$
(64)	-1	4	-6	4	$2(35)^{1/2}$

actions between the vector octuplet and the pseudoscalar octuplet. Let us suppose that for the generation of vector mesons, graphs a, b, and c in Fig. 1 are the important ones which involve only bosons, and that

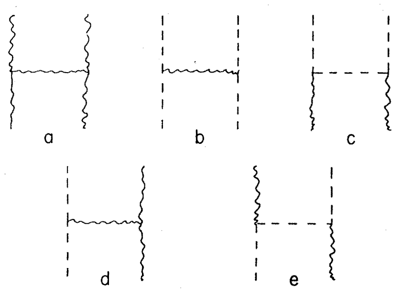

FIG. 1. Wavy lines represent vector mesons, and dashed lines represent pseudoscalar mesons. Graphs a, b, and c are assumed to contribute strongly to the formation of vector mesons, and graphs d and e to the formation of pseudoscalar mesons.

$1d$ and $1e$ are the primary bosonic graphs for generation of the pseudoscalar mesons. These five graphs have in common the feature that all the vertices involve the same antisymmetric quantity F_{ijk}.[2] Therefore, as long as only these graphs are taken into account, the entire discussion of I (which was based on $1a$) applies here with very little change. The main difference is, that in the discussion of the pseudoscalar states, additional types of admixtures can occur because the two constituents are not identical, so that the treatment of the vertex corrections must be modified. However, if we assume that the deviations in the two supermultiplets have the same sign and a similar magnitude, there will be little change in the effects on the vertices of the mass perturbations because the extra admixtures are only produced by differences in the dynamical behavior of the two kinds of bosons. In other words, similar

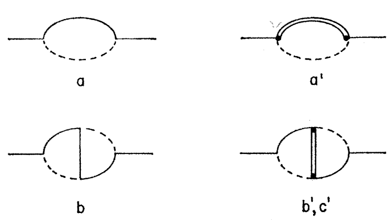

FIG. 2. Graphs characterizing the calculation of perturbations in the baryon states. In Figs. 2–4, and 5, solid lines denote the baryons, double lines the excited baryon states, and dashed lines either pseudoscalar or vector mesons.

[2] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).

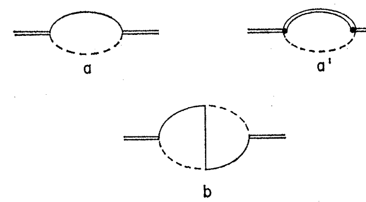

FIG. 3. Graphs characterizing perturbations of the B^* masses.

deviations in the two octuplets would be a self-consistent possibility; for the (8) perturbation, this choice is certainly optimum for self-generation, but for the (27) perturbations it is not self-generating so another possibility might be somewhat better.

The meson-baryon couplings involve "mixing parameters," and the results depend very strongly on the values chosen. For the pseudoscalar coupling, arguments[2–5] have been given in support of a mixing angle near 33°, which we use in constructing tables of numerical results. The vector mesons have two couplings, which are unknown and might have quite different angles. We assume for numerical definiteness that the effective average (ϕ) can be well approximated by 33°. This value is, at least, not ruled out by existing considerations.[6–8] Since we consider only the case that the vector and pseudoscalar mesons have similar mass deviations, we then do not need to distinguish between the contributions of the two kinds of mesons; the results are independent of the relative importance of the two kinds of mesons in the various graphs. Of

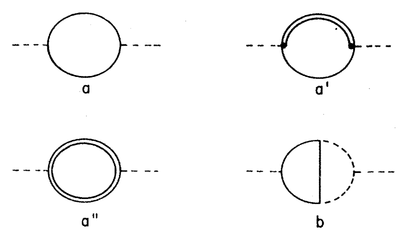

FIG. 4. Graphs which characterize those perturbations of the meson masses which involve virtual baryon–antibaryon pairs.

course, the results are more likely to be in accordance with reality when the pseudoscalar mesons give the main contribution.

II. DISCUSSION OF FIRST ORDER PERTURBATIONS

The perturbations upon the particle masses that we are considering are described by the graphs in Figs. 2–4. A shorthand notation has been used to describe these perturbations. Figure 2 describes the $\frac{1}{2}(+)$ octuplet (B), Fig. 3 the $\frac{3}{2}(+)$ decuplet (B^*), and Fig. 4 the contribution of virtual baryon pairs to the boson

[3] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).
[4] S. Glashow and A. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).
[5] Pekka Tarjanne, Bull. Am. Phys. Soc. **8**, 45 (1963).
[6] J. J. Sakurai, (to be published), and private communication.
[7] J. Kalckar, Phys. Rev. **131**, 2242 (1963), and private communication.
[8] A. W. Martin and K. C. Wali (private communication).

TABLE III. The values of $K_g(i,j)$ for (27) perturbations. The starred entries occur with a double weight in $k(i,j)$.

Graph	B	M	B^*	
$2a$	0.04			
$3a$			0.39	
$3b$			0.12	B
$4a$		0.04*		
$4a'$		−0.07		
$4b$		−0.07		
Ia		−0.33*		
Ib		+0.56		
$2a$	0.04			
$2a'$	−0.07			M
$2b$	−0.07			
$3a$			0.39	
$3a'$			−0.39	
$2a'$	0.39			
$2b'$	0.12			
$3a'$			0.33	B^*
$4a'$		0.39		
$4a''$		−0.39*		

states. The graphs labeled "a" refer to mass changes of the constituent particles (in the legs of ladder graphs); those labeled "b," to mass changes of the exchanged particles (the ladder rungs), and the "c" graphs, to vertex perturbations. This corresponds to the notation of I; we shall refer to the corresponding graphs of I by Ia, etc. We do not otherwise indicate explicitly in Figs. 2–4 the site of the perturbing influence. Primes indicate the presence of internal B^* lines.

In considering dissymmetries of some given transformation type, we denote by $A(i)$ the coefficient of the normalized tensor operator $D(i)$ describing the dissymmetry of the supermultiplet i [there are two values of i for the (8) dissymmetry of the baryons, corresponding to symmetric and antisymmetric tensors]. The first-order perturbation of the masses can be expressed as

$$A'(i) = \sum_j k(i,j) A(j). \qquad (1)$$

Self-generation, in accordance with the discussion in I, corresponds to $A'(i) \approx A(i)$, or to $k(i,j)$ having an eigenvalue very nearly equal to unity.

The contribution $k_g(i,j)$ of a graph g of the a or b type we write as $k_g(i,j) = \alpha_g(i,j) K_g(i,j)$, where $\alpha_g(i,j)$ is defined to be independent of the transformation type, and represents the effect of perturbing the mass of a given internal line [assuming the masses of all the particles in the supermultiplet (j) are perturbed equally], and $K_g(i,j)$ denotes the sum over the contributions of the individual charge and hypercharge states to the effective perturbation. In other words, for a (1) perturbation, we have $k_g(i,j) \equiv \alpha_g(i,j)$ for a or b graphs, whereas for c graphs, $k_g(i,j) = 0$.

The calculation of the $\alpha_g(i,j)$ depends on properties of the states which we do not know very well, but we are able to exploit the scaling property of the self-consistency equations to learn something about these parameters. As pointed out in I, we must be sure that our approximations are consistent with the principle that there is no fundamental length (except the masses of the particles themselves) which enters into the calculation, so that if all input particle masses are multiplied by a factor, the calculated masses are necessarily multiplied by the same factor. This implies that for a (1) perturbation, $k(i,j)$ must have one eigenvalue which is exactly equal to unity. The corresponding eigenvector $A(i)$ is given by the mass ratios. Although we do not know the actual contributions of the different graphs, if the values of the $K_g(i,j)$ of the important graphs do not differ too much, the uncertainties will cancel out.

The actual evaluations of the sums over isotopic spin and hypercharge states are rather tedious, and a discussion is relegated to the Appendix. The numerical results are given in Tables III and IV for (8) and (27) perturbations.

It was found in I that a major contribution leading to the possibility of self-generation of the (8) perturbation comes from the vertex corrections. It was possible for us to estimate the magnitude of the vertex contribution because there was only one kind of supermultiplet which could be admixed into the octuplet, and in this other supermultiplet the ladder-approximation potential vanished. It is also possible to obtain an estimate here of the admixture of states into the decuplet because there is a good model for these states which also predicts the relative potentials in the other states which might be admixed, and standard perturba-

TABLE IV. The values of $K_g(i,j)$ for (8) perturbations. The starred entries occur with a double weight in $k(i,j)$.

Graph	$B(-)$	$B(+)$	M	B^*	
$2a$	0.50	−0.46			
$3a$				0.70	
$3b$				0	$B(-)$
$4a$			0.46*		
$4a'$			0.45		
$4b$			0.25		
$2a$	−0.46	−0.06			
$3a$				0.31	
$3b$				−0.21	$B(+)$
$4a$			−0.06*		
$4a'$			−0.40		
$4b$			−0.15		
Ia			0.50*		
Ib			0		
$2a$	0.46	−0.06			M
$2a'$	0.45	−0.40			
$2b$	0.25	−0.15			
$3a$				0.31	
$3a'$				0.47	
$2a'$	0.70	0.31			
$2b'$	0	−0.21			
$3a'$				0.75	B^*
$4a'$			0.31		
$4a''$			0.47*		

TABLE V. The values of $K(i,j)$ which arise from perturbation of B^* coupling constants. The table shows how the totals are comprised of contributions from individual configurations admixed into the decuplet (3,0). Ratios of the admixture coefficients were obtained by applying perturbation theory to Eq. (3).[a]

Admixed B^* component	Induced splitting of baryon masses (8)+	(8)−	(27)	Perturbation
(2,2)	0	1.68	0	M
(0,3)	0	0	0.176	
$(1,1)_A$	−0.19	0	−0.016	
$(1,1)_S$	0	−0.07	0	
Total	−0.19	1.61	0.16	
(2,2)	0	−1.68	0	B
(0,3)	0	0	0.176	(8)+
$(1,1)_A$	−0.19	0	−0.016	or
$(1,1)_S$	0	0.07	0	(27)
Total	−0.19	−1.61	0.16	
(2,2)	0	1.25		
(0,3)	0	0		B
$(1,1)_A$	0	0		(8)−
$(1,1)_S$	0	0.08		
Total	0	1.33		

[a] Only ratios of the entries given are significant. The observed M and B (8) dissymmetries are shown, by Eq. (6), to be almost equally effective in inducing (2,2) admixtures.

tion theory can be used. The graphs in Fig. 5 describe these vertex perturbations; the results of the calculation, showing the influence of the perturbed vertices on the octuplet masses, are given in Table V. On the other hand, our dynamical model of the baryon octuplet is not reliable enough for us to use in a calculation of the admixtures into these states, so we shall have to rely on a heuristic argument in discussing their effect.

We observe that the perturbed coupling constants (resulting from admixture of representations) will be increased when the coupled particles have small masses. Therefore, the effect of the c graphs will correspond, approximately, to an average of the graphs a, a', and b which are associated with the three lines leading into the vertex; roughly speaking, $k_c(i,j)\approx[k_a(i,j)+k_{a'}(i,j)+k_b(i,j)]\times$constant. A similar result holds for the calculation of the perturbed vertex itself; an additional amplifying effect arises from the dependence of the vertex corrections on the input vertex perturbations. We infer, therefore, that if the average of the a, a' and b contributions is very small, k_c will also be very small. If, on the other hand, the average is positive and not too small, k_c might be quite large as a result of the amplification. These observations agree with the result of calculation in I, but here we shall not be able to express them in a quantitative form.

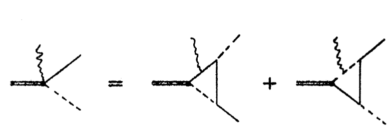

FIG. 5. These graphs symbolize the calculation of the B^* vertex perturbations. The wavy lines represent the perturbations.

We see from Table III that the $K_g(i,j)$ are very small for (27) perturbations—except for graphs Ia and Ib, and a few graphs (involving B^* particles) which are probably not very important. Moreover, the signs of a number of the terms are such as to oppose self-generation. In Table IV on the other hand, the entries are consistently larger, and have signs proper for self-generation [except for the one very small entry from graph $2a$, and the terms, relating $A(B^*)$ and $A(B+)$, from graphs $2a'$ and $3a$]. Assuming that self-generation occurs, one requires that $A(M)$, $A(B-)$ and $A(B^*)$ should have the same sign, and that $A(B+)$ should have the opposite sign. (These are, in fact, the relative signs observed in these supermultiplets.) In order for self-generation to actually occur, a larger contribution to $k(i,j)$ must be obtained from the c graphs than from the a and b graphs together, but such a large contribution is not unreasonable.[9]

Table V shows that the combined effect of graph $2c$ and the graphs in Fig. 5 is very small for a (27) perturbation. We also observe, however, that they provide an important mechanism for the self-generation of (8) perturbations. The observed baryon and meson-mass splittings combine to produce a large matrix element between states from the (2,2) and (3,0) representations; admixture of (2,2) states is again favored by their attractive potential. Finally, especially when the meson-baryon coupling is characterized by an angle θ near to 33°, this admixture produces a strong splitting between the Ξ and N masses. In the next section, we shall attempt to make a quantitative estimate of the admixture (using experimental mass values); for the N^*, the admixture is found to be so large that the N^* is almost exclusively coupled to $(N\pi)$ states, and is coupled very weakly to (ΣK) states. As a result, the B^*-exchange potential nearly vanishes in the Ξ state, and in the N state is about twice as strong as it would be in the case of pure SU_3 symmetry.

Self-generation of a small (27) perturbation appears to be ruled out by the considerations above. On the other hand, self-generation of a small (8) perturbation, while by no means established, seems to be consistent with a variety of plausible internal structures for the particles we have considered. In other words, we might say that a number of *mechanisms* which might lead to

[9] There is a further conceivable dissymmetry of the baryon octuplet which consists of a combination of (3,0) and (0,3) tensors. This type of dissymmetry retains neither charge independence nor charge symmetry, but the baryon masses do remain invariant under the rotation subgroup of the Weyl reflections of the SU_3 roots. We calculate from graph $2a$ that $K=-0.28$, and since this dissymmetry could occur in neither the meson octuplets nor the B^* decuplet, it could not be self-generating. The question has been raised whether discrete strong-interaction symmetries might necessarily be transformed by gauge invariances (such as charge, hypercharge, baryon number) into symmetries associated with semisimple Lie groups [S. Weinberg (unpublished)]. We have here an example (some others appear in I) in which the elimination of the possibility of a truly discrete residual symmetry is not achieved through kinematical arguments, but requires the examination of dynamical self-consistency.

self-generation have been exhibited, even though we have not been able to perform the calculations needed to prove that these mechanisms have just the right combined strength to lead to the dissymmetries observed.

We have remarked that the relative signs of the $A(i)$ are given correctly by our model. It is hard to estimate the ratios more precisely, except for $R=-A(B-)/A(B+)$, in which many of the uncertainties in the internal structure of the baryons tend to cancel.[10] The experimental value is $R=4.5$. Since the entries in Table IV suggest $R\sim 2-3$, the c graphs must influence this ratio strongly. In fact, Table V shows that the B^*-exchange graph contributes much more to $A(B-)$ than to $A(B+)$. If B^* exchange were to contribute about -200 MeV to the nucleon mass, this would be consistent with all other contributions giving $R\sim 2-3$.

The mass splittings in the $\frac{5}{2}(+)$ excited baryon octuplet should be much like those in the $\frac{1}{2}(+)$ octuplet. However, the splittings in the $\frac{3}{2}(-)$ octuplet depend on the sort of model that is used for these states. If Peierls' mechanism,[11,12] is the main effect, then the B' splittings will not be related in a simple way to the B splittings, and, also, one would not even expect first-order perturbation theory to give a reasonable approximation, as discussed by Tuan.[12] If the model discussed in Ref. 2 were more nearly correct, one would expect the coefficient $A(B-)$ to be very small in the $\frac{3}{2}(-)$ octuplet; the splitting of the two $Y=+1$ Regge trajectories ought to be nearly equal to the $\Xi-N$ mass difference, and the splitting of the two $Y=-1$ trajectories ought to nearly vanish. We should like to comment in addition only that further study of mass splittings will be an extremely useful tool for investigating the inner dynamical structures of these resonances.

Sakurai[6] has suggested a model in which the "φ–ω mixing" is taken as fundamental and the baryon mass differences arise from graph 2a. In discussing φ–ω mixing, we take the boson lines to represent any of the *nine* vector mesons, and the perturbation to be the off-diagonal mass term coupling the singlet to the octuplet. Since this mixing effect can only occur as an (8) perturbation, it gives an additional means of favoring self-generation of (8) perturbations rather than (27) perturbations. We denote by μ the off-diagonal φ–ω mass. The perturbation of the baryon masses arising from inserting μ into graphs 2a and 2b is:

$$A(B-)=\alpha_1\mu\sin\phi,\qquad A(B+)=\alpha_1\mu\cos\phi, \tag{2a}$$

$$A(B-)=-\tfrac{1}{2}\alpha_2\mu,\quad\text{and}\quad A(B+)=-\alpha_2\mu\sin 2\theta \tag{2b}$$

[taking into account in (2b) only the pseudoscalar mesons]. The off-diagonal mass μ can in turn be generated by graphs 4a and 4b, which have a dependence on θ and ϕ which is identical to that of 2a and 2b.[13] The coefficients α_1 and α_2 depend on the internal structure of the baryons, as well as on the coupling constants, but if $\phi\approx\theta$, as we have been assuming so far, physical arguments can be given for their having the same sign.

If it is true that the vector meson couplings can be well described by a single effective ϕ which is close to θ (and to 33°), the effects of these two graphs tend to cancel, and, moreover, the observed sign of R could not arise from Eqs. (2a and b) above except through fortuitous cancellations. The form of Eqs. (2a and b) would seem, therefore, to preclude an important role for φ–ω mixing in the self-generation of an (8) dissymmetry; also, Tables IV and V suggest that it wouldn't be needed. On the other hand, if this picture of the vector meson couplings were wrong, then we wouldn't get quite so much regeneration from the other perturbations (because Table IV would apply only to the pseudoscalar mesons), so the increased possibility of a strong φ–ω mixing effect might, in this case, be important for self-generation. Self-generation, in other words, can possibly be reconciled with a variety of situations, but Sakurai's φ–ω mixing mechanism is, unfortunately, somewhat more complicated than it appears to be at first sight.[14]

III. HIGHER ORDER PERTURBATIONS IN THE DECUPLET

The first question about higher order perturbations which comes to mind is, why do the mesons obey the Gell-Mann–Okubo formula better when the squares of the masses are used? Evidently, certain higher order perturbation terms cancel more effectively when expressed this way, but we are not able to give any reason why this should be so. A second question is why the formula should work so well, despite such enormous first-order perturbations. Again, there must be cancelations among the (27) deviations induced in higher orders. Some tendency toward such cancelation

[10] We should like to point out here some respects in which the discussion of baryon masses given in this paper is an improvement over that given in Ref. 2. First, we are looking here for a unified picture of all mass deviations. Second, in Ref. 2, only graph 2a was considered. Third, the estimates of the parameters α_{2a} given in Ref. 2 did not emphasize the scaling property of the self-consistency equations, and are undoubtedly overestimates. The decreased influence of graph 2a we expect to be partly compensated by vertex corrections having similar properties.

[11] R. F. Peierls, Phys. Rev. Letters **6**, 641 (1961).

[12] S. F. Tuan, Bull. Am. Phys. Soc. **8**, 364 (1963), and private communication.

[13] Another interesting mechanism for inducing φ–ω mixing has been described recently by A. Katz and H. Lipkin (unpublished). The Katz-Lipkin mechanism involves boson couplings which go beyond those in Fig. 1. It is clear that understanding of the origin of φ–ω mixing requires a detailed model of these mesons, in particular, of the relative importance of $\bar{B}B$ and bosonic components. Although very little is known about this question, some preliminary work has been done by Kalckar (see Ref. 7).

[14] Another possibility, which can not yet be ruled out, is that the contribution from φ–ω mixing to the baryon mass splittings actually opposes self-generation, but serves to compensate for another perturbation (for example, from B^* exchange) which is too strong.

was reported in I, and seems to arise also in the calculations reported here on the decuplet masses. Still another question might be concerned with the reasonableness of discussing self-generation by first-order perturbations, in the face of some effects which perturbation theory is incapable of describing.

As an example, consider the off-diagonal φ–ω mass μ, which is believed to be so large compared with the difference in the masses of these particles,[6,7,15] that the physical ω and φ are admixtures, with nearly equal coefficients, of the unperturbed singlet and octuplet states. The influence of the mass matrix M on virtual vector mesons is described through a propagator $\Lambda(q)/(q^2-M^2)$. The physical particles correspond to the vanishing of the denominator, where the off-diagonal term has a large effect. For virtual vector mesons, however, q^2 may even be negative, in which case, considering the magnitude of μ, it is certainly valid to expand in powers of μ and keep only the first few terms. This shows that there is no necessary inconsistency between nonperturbative influences on certain physical states, and the use of perturbation theory to describe these effects when the particles occur virtually in the internal structure of other particles.

A similar situation arises from the fact that the energy of a resonance is a nonanalytic function of the input parameters when the energy coincides with a threshold.[16] In particular, the width of a resonance is obviously nonanalytic; this is usually taken into account by first estimating the real part of the energy and the coupling constants (or reduced widths), and then introducing phase space factors. Nonanalytic threshold terms, however, remain in the energy and coupling constants; except for S waves, they are generally small, and in an effective range formula of the Chew–Low type, the threshold terms are omitted.[17] Nonanalyticity is therefore not a practical problem; fortunately, it does not lead to theoretical difficulties either, because the nonanalyticity is cured when the resonance is "sufficiently virtual."[18]

Even if we disregard these threshold phenomena, however, the B^* coupling constants may be expected to be perturbed strongly from the SU_3 values, because, according to our picture of these particles, the kinetic energies of their constituents are no larger than some of the mass differences. Therefore, we need to examine these states without being limited by perturbation theory. The problem of calculating B^* energies and coupling constants from the dispersion relations has been discussed by Martin and Wali[3] and by Capps,[19] but since such calculations do not lend themselves to easy visualization of the results, we shall represent the main effects by a simplified model.

In our model, we determine the energy E from the eigenvalue problem

$$E\psi=(K+V)\psi, \tag{3}$$

where the components of the eigenvector ψ are proportional to the B^* coupling constants. For K, we write

$$K=(M^2+p^2)^{1/2}+(m^2+p^2)^{1/2}, \tag{4}$$

where M and m are the observed masses of the baryon and the pseudoscalar meson which occur in a given channel, and p is an average relative momentum; we assume that the B^* states are all characterized by the same radius and hence by the same value of p. We take for the potential

$$V=V_B+C, \tag{5}$$

where V_B is proportional to the baryon-exchange pole term, and C is an adjustable constant, independent of the channel, which we choose so as to make the center of gravity of the decuplet appear at the correct energy.

The perturbation of V_B is uncertain, but not believed to be large, so we take the exact SU_3 values, as calculated with $\theta=33°$. Since it is of some interest to examine the influence of a pure (8) perturbation in K, we make the following approximation:

$$K=(M_0^2+p^2)^{1/2}+(m_0^2+p^2)^{1/2} +M_0(M_0^2+p^2)^{-1/2}\Delta M+\tfrac{1}{2}(m_0^2+p^2)^{-1/2}\Delta(m^2), \tag{6}$$

where ΔM and $\Delta(m^2)$ denote the eightfold part of the deviations from the average values M_0 and m_0^2. Since these approximations cannot be expected to be very accurate, the results must be interpreted as suggesting only the general outlines of the actual situation.

The energies of the N^*, Y^*, and Ξ^* can be fit approximately (but not perfectly) by a variety of values for p, C, and the proportionality constant in V_B. We give in Table VI some data for a fit in which C is

TABLE VI Properties of $j=\frac{3}{2}(+)$ states (B^*).

		Input data		
Parameter	p	C	$V_B(3,0)$	$V_B(2,2)$
Value (MeV)	500	−45	−410	−140
		Results		
State	$N_{3/2}^*$	Y_1^*	$\Xi_{1/2}^*$	Ω_0
Energy (MeV)	1240	1391	1529	1658
Weights of component states				
(3,0)	0.78	0.86	0.93	1
(2,2)	0.22	0.14	0.07	0
pionic	0.91	0.59	0.29	0

[15] J. J. Sakurai, Phys. Rev. Letters **9**, 472, (1962).

[16] R. J. Oakes and C. N. Yang, Phys. Rev. Letters **11**, 174 (1963).

[17] An alternative approach to this problem has been discussed by M. Ross (unpublished). The general conclusions obtained by Ross are similar to ours.

[18] That is, a two-particle Green's function, at an energy below all thresholds, may remain an analytic function of its parameters even for parameter values at which a pole in the energy variable coincides with a threshold.

[19] R. E. Capps, Nuovo Cimento **13**, 1208 (1963).

relatively small, and p lies in a reasonable range.[20] The residual discrepancies with the observed energies are only a few percent of the amounts by which the energies have been displaced by the input mass deviations. (Some fits with $C=0$ are nearly as good.) The energy of the Ω is somewhat sensitive to the values chosen for the parameters (assuming a fit to the other energies).

A general characteristic of this model is that there are enormous admixtures of states from the (2,2) configuration into the (3,0) states. The admixtures of other states are considerably smaller than first-order perturbation theory would suggest, and, in fact are quite negligible. Despite the impossibility of describing the eigenstates by first-order perturbation theory, the energy levels obey the Gell-Mann–Okubo rule quite well, as a result of cancellations among higher order terms.

From the eigenvector obtained for the Y_1^* we derive a considerable enhancement of $(\Lambda\pi)$ decays, and also some inhibition of $(\Sigma\pi)$ decays:

$$\Gamma(\Sigma\pi)=0.07\Gamma(\Lambda\pi). \quad (7)$$

The probability for virtual dissociation of a given B^* into a pion and a baryon is found in the last row of Table VI. In the N^*, the SU_3 description is considerably less accurate than the xenophobic description (disregard of strange particles), but SU_3 gives a better approximation to the strange resonances. The fact that the N^* is coupled almost exclusively to the $(N\pi)$ channel is the basis for our remarks in the previous section about the strength of the B^*-exchange perturbations upon the baryon masses. As a result of the large modifications in the B^* coupling constants, however, the use of a first-order theory to describe these effects has only qualitative significance.

IV. SUMMARY

Perturbations from SU_3 symmetry have been examined for self reproducibility in a model containing pseudoscalar- and vector-meson octuplets, a $j=\frac{1}{2}(+)$ baryon octuplet, and $j=\frac{3}{2}(+)$ baryon decuplet. It has been shown that the contributions of perturbations in individual charge and hypercharge states combine in such a way as to tend to make the model less stable against small (8) perturbations than against other kinds of perturbations. In Paper I, this same feature had been observed in a model containing only vector mesons, and was shown there to provide a mechanism for the existence of isotopic spin symmetry, as well as for the Gell-Mann–Okubo mass rule.

Needless to say, the model is still oversimplified, and the present qualitative estimates of the magnitudes of the induced perturbations must be replaced by precise calculations before definite conclusions can be drawn.

APPENDIX

The $K_g(i,j)$, for g belonging to the class a, are analogs for SU_3 of the Racah coefficients, or $6-j$ symbols, which are familiar from the theory of angular momentum. For g of the class b, the K_g are analogs of $9-j$ symbols. Tables of these quantities do not yet exist, so we have had to calculate for ourselves the ones we needed. Nevertheless, consideration of the symmetries of the coefficients, which we now describe briefly, has been of great utility.

Denote by capital letters A, B, $\cdots$ the multiplicities of irreducible representations, and by the corresponding lower-case letter a state from the representation. Sum over repeated indices. Define Clebsch-Gordan coefficients C_{abc} normalized symmetrically:

$$C_{abc}C_{abc}=ABC. \quad \text{(A1)}$$

Consider a mass perturbation $(CD)^{-1/2}C_{cdx}\Delta_x$ in some (a) graph g. The induced mass change (apart from dynamical factors) is

$$(CDE)^{-1}C_{ace}C_{cdx}C_{bde}\Delta_x=L_g(AB)^{-1/2}C_{abx}\Delta_x. \quad \text{(A2)}$$

In the inverse graph g' (e.g., related to g as $4a'$ is to $2a'$), $(AB)^{-1/2}C_{abx}\Delta_x$ is the input, and the induced perturbation is

$$(ABE)^{-1}C_{ace}C_{abx}C_{bde}\Delta_x=L_{g'}(CD)^{-1/2}C_{cdx}\Delta_x. \quad \text{(A3)}$$

Use of (A1) leads to

$$\begin{aligned}(AB)^{-1/2}L_g&=(CD)^{-1/2}L_{g'}\\ &=(ABCDEX)^{-1}C_{abx}C_{ace}C_{bde}C_{cdx}. \quad \text{(A4)}\end{aligned}$$

If we disregard configuration mixing, as in most of this paper, $A\equiv B$ and $C\equiv D$. A special case is $X=1$; we then have $C_{abx}=A^{1/2}\delta_{ab}$, and (A4) reduces to $A^{-1}L_g(1)=C^{-1}L_{g'}(1)=(AC)^{-1/2}$. Since we defined $K_g=L_g/L_g(1)$, we have $K_{g'}\equiv K_g$. This symmetry obviously holds for the b graphs, too. Further symmetries are apparent from (A4), but require attention to the symmetries of the Clebsch-Gordan coefficients to determine the sign.

In the evaluation of (A4), we used two different techniques. First, if a sufficient number of the representations were eightfold, so that at least two of the C's were linear combinations of the F and D matrices, known properties of these matrices (as described in Ref. 2, for example) could be used to obtain an immediate answer. (This method also works for b graphs, if four of the C's are of this type.) Illustrations are given in I. For calculations involving the decuplet, we found simplest the prosaic method of evaluating (A2) explicitly, using tabulated values of the Clebsch-Gordan coefficients.[21]

[20] We are grateful to J. M. Wang for helping us with these numerical calculations.

[21] P. Tarjanne, Carnegie Institute of Technology, Reports NYO-9290 and NYO-9290-A (unpublished).